Riemann integration

Partition (Division) of an integral.

Let $[a,b]$ be a closed bounded integral.
By a partition P of $[a,b]$ we mean a finite ordered subset $\{x_0, x_1, x_2 \text{---} x_n\}$ of $[a,b]$ such that $a = x_0 < x_1 \text{---} x_{n-1} < x_n = b$

Such a partition P is often denoted by $P = \{a = x_0, x_1, \ldots x_n = b\}$

Upper and Lower (Riemann) Sums. (or Darboux Sums)

Let f be a bounded function defined on a closed bounded interval $[a,b]$, $a<b$

Page No.:
Date: / /

Let $P = \{a = x_0, x_1, x_2 \text{ ---- } x_n = b\}$ be any partition of $[a, b]$ and let

δ_r = the length of the sub interval $[x_{r-1}, x_r]$; $r = 1, 2, 3$ ----

Since f is bounded in $[a,b]$, therefore it is necessary by bodd in each sub-interval δ_r.
Let M_r denote the lub and m_r, denote the g.l.b (infimum) of f in $[x_{r-1}, x_r]$; $r = 1, 2, 3$ ---- n

We form the sums $U(P, f) = \sum_{r=1}^{n} M_r \delta_r$

and $L(P, f) = \sum_{r=1}^{n} m_r \delta_r$. — (1)

Then $U(P, f)$ is called the upper Riemann sum of f on $[a, b]$ correspon-ding to the partition P. And $L(P,f)$ is called the lower Riemann sum of f corresponding to the Partition P.

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